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Spacetime Non-Commutativity Corrections to the Cardy-Verlinde Formula of Achúcarro-Ortiz Black Hole

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In this letter we compute the corrections to the Cardy-Verlinde formula of Achúcarro-Ortiz black hole, which is the most general two-dimensional black hole derived from the three-dimensional rotating Banados-Teitelboim-Zanelli black hole. These corrections stem from the space non-commutativity. We show that in non-commutative case, non-rotating Achúcarro-Ortiz black hole in contrast with commutative case has two horizons.

KEY WORDS: Cardy-Verlinde formula; Achúcarro-Ortiz black hole; space non-commutativity; horizon.

1. INTRODUCTION

The Cardy-Verlinde formula proposed by Verlinde (2000), relates the entropy of a certain CFT with its total energy and its Casimir energy in arbitrary dimensions. Using the $\mathrm{AdS}_d/\mathrm{CFT}_{d-1}$ (Maldacena, 1998) and $\mathrm{dS}_d/\mathrm{CFT}_{d-1}$ correspondences (Strominger, 2001a,b; Spradlin *et al.*, 2001), this formula has been shown to hold exactly for different black holes (see for example Balasubramanian *et al.*, 2002; Danielsson, 2002; McInnes, 2002; Myung, 2001; Nojiri and Odintsov, 2001a,bc; Ogushi, 2002; Setare and Mansouri, 2003; Setare, 2002; Shiromizu *et al.*, 2001; Setare and Altaie, 2003). In this paper we consider the Cardy-Verlinde formula of the Achúcarro-Ortiz black hole which is a two-dimensional black hole derived from the three-dimensional rotating Bañados, Teitelboim and Zanelli (BTZ) black hole.

In 1992 Bañados, Teitelboim and Zanelli (BTZ) Bañados *et al.* (1992, 1993) showed that $(2+1)$-dimensional gravity has a black hole solution. This black hole is described by two parameters, its mass M and its angular momentum (spin) J. It is locally anti-de-Sitter space and thus it differs from Schwarzschild and Kerr solutions in that it is asymptotically anti-de-Sitter instead of flat spacetime.

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Additionally, it has no curvature singularity at the origin. AdS black holes, which are members of the two-parametric family of BTZ black holes, play a central role in AdS/CFT conjecture (Maldacena, 1998) and also in brane-world scenarios (Randall and Sundrum, 1999a,b). Specifically AdS(2) black hole is most interesting in the context of string theory and black hole physics (Birmingham *et al.*, 1997; Maldacena *et al.*, 1999). Although the possibility of describing 2D black holes by means of a CFT has been widely investigated (Cadoni and Mignemi, 1999a,b; Strominger, 1999; Cacciatori *et al.*, 2000; Cadoni and Cavaglià, 2001a,b; Cadoni and Carta, 2001), it is not completely clear if it is always possible to mimic the gravitational dynamics of the 2D black hole through a CFT. However, in some cases and/or for generic black holes in particular regimes, CFTs have been shown to give a good description, this is in particular true for black holes in AdS space.

Black hole thermodynamic quantities depend on the Hawking temperature via the usual thermodynamic relations. The Hawking temperature undergoes corrections from many sources:the quantum corrections (Das *et al.*, 2002; Lidsey *et al.*, 2002; Setare, 2003, 2004a), the self-gravitational corrections (Keski-Vakkuri and Kraus, 1996; Parikh and Wilczek, 2000; Setare and Vagenas, 2004), and the corrections due to the generalized uncertainty principle (Setare, 2004b).

In this letter we concentrate on the corrections due to the space non commutativity. Recently there has been considerable interest in the possible effects of the non commutative space (Seiberg and Witten, 1999).

By considering a black hole as quantum state instead of a classical object ('tHooft, 1985, 1996; Yan and Bai, 2004) and according to quantum mechanics principle, one can conclude its energy and its corresponding conjugate time can not be simultaneously measured exactly. The energy of states should approximately be the hole's gravitational energies measured at the region of the horizon. In other hand the gravitational energies are quasilocal, in the Schwarzschild black hole case, this quasilocal energy is proportional to the radius of event horizon. Therefore the uncertainty relation between energy and time in the event horizon region lead that the radial coordinate is noncommutative with time at the horizon.

2. ACHÚCARRO-ORTIZ BLACK HOLE

The black hole solutions of Bañados Teitelboim and Zanelli (Bañados *et al.*, 1992, 1993) in $(2 + 1)$ spacetime dimensions are derived from a three dimensional theory of gravity

$$S = \int dx^3 \sqrt{-g} \left({}^{(3)}R + 2\Lambda \right) \tag{1}$$

with a negative cosmological constant ($\Lambda = 1/l^2 > 0$).

The corresponding line element is

$$ds^2 = -\left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)dt^2 + \frac{dr^2}{\left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)} + r^2\left(d\theta - \frac{J}{2r^2}dt\right)^2 \tag{2}$$

It is obvious that there are many ways to reduce the three dimensional BTZ black hole solutions to the two dimensional charged and uncharged dilatonic black holes (Achúcarro and Ortiz, 1993; Lowe and Strominger, 1994). The Kaluza-Klein reduction of the (2 + 1)-dimensional metric (2) yields a two-dimensional line element:

$$ds^2 = -g(r)dt^2 + g(r)^{-1}dr^2 \tag{3}$$

where

$$g(r) = \left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right) \tag{4}$$

with M the mass of the two-dimensional Achúcarro-Ortiz black hole, J the angular momentum (spin) of the afore-mentioned black hole and $-\infty < t < +\infty$, $0 \leq r < +\infty$, $0 \leq \theta < 2\pi$.

The outer and inner horizons, i.e. r_+ (henceforth simply black hole horizon) and r_- respectively, concerning the positive mass black hole spectrum with spin ($J \neq 0$) of the line element (2) are given as

$$r_\pm^2 = \frac{l^2}{2}\left(M \pm \sqrt{M^2 - \frac{J^2}{l^2}}\right) \tag{5}$$

and therefore, in terms of the inner and outer horizons, the black hole mass and the angular momentum are given, respectively, by

$$M = \frac{r_+^2}{l^2} + \frac{J^2}{4r_+^2} \tag{6}$$

and

$$J = \frac{2\,r_+ r_-}{l} \tag{7}$$

with the corresponding angular velocity to be

$$\Omega = \frac{J}{2r^2}\ . \tag{8}$$

The Hawking temperature T_H of the black hole horizon is (Kumar and Ray 1995)

$$T_H = \frac{1}{2\pi r_+}\sqrt{\left(\frac{r_+^2}{l^2} + \frac{J^2}{4r_+^2}\right)^2 - \frac{J^2}{l^2}}$$
$$= \frac{1}{2\pi r_+}\left(\frac{r_+^2}{l^2} - \frac{J^2}{4r_+^2}\right). \tag{9}$$

The area $\mathcal{A}_{\mathcal{H}}$ of the black hole horizon is

$$\mathcal{A}_{\mathcal{H}} = \sqrt{2}\pi l\left(M + \sqrt{M^2 - \frac{J^2}{l^2}}\right)^{1/2} \tag{10}$$

$$= 2\pi r_+ \tag{11}$$

and thus the entropy of the two-dimensional Achúcarro-Ortiz black hole, if we employ the well-known Bekenstein-Hawking area formula (S_{BH}) for the entropy (Bekenstein, 1973, 1974; Hawking, 1976), is given by

$$S_{BH} = \frac{1}{4\hbar G}\mathcal{A}_{\mathcal{H}} \ . \tag{12}$$

Using the BTZ units where $8\,\hbar G = 1$, the entropy of the two-dimensional Achúcarro-Ortiz black hole takes the form

$$S_{BH} = 4\pi r_+ \ . \tag{13}$$

3. CARDY-VERLINDE FORMULA

In a recent paper, Verlinde (2000) propound a generalization of the Cardy formula which holds for the $(1 + 1)$ dimensional Conformal Field Theory (CFT), to $(n + 1)$-dimensional spacetime described by the metric

$$ds^2 = -dt^2 + R^2 d\Omega_n \tag{14}$$

where R is the radius of a n-dimensional sphere.

The generalized Cardy formula (hereafter named Cardy-Verlinde formula) is given by

$$S_{CFT} = \frac{2\pi R}{\sqrt{ab}}\sqrt{E_C\,(2E - E_C)} \tag{15}$$

where E is the total energy and E_C is the Casimir energy. The definition of the Casimir energy is derived by the violation of the Euler relation as

$$E_C \equiv n\,(E + pV - TS - J\Omega) \tag{16}$$

where the pressure of the CFT is defined as $p = E/nV$. The total energy may be written as the sum of two terms

$$E(S, V) = E_E(S, V) + \frac{1}{2}E_C(S, V) \tag{17}$$

where E_E is the purely extensive part of the total energy E. The Casimir energy E_C as well as the purely extensive part of energy E_E expressed in terms of the radius R and the entropy S are written as

$$E_C = \frac{b}{2\pi R}S^{1-\frac{1}{n}} \tag{18}$$

$$E_E = \frac{a}{4\pi R}S^{1+\frac{1}{n}} \quad . \tag{19}$$

After the work of Witten on AdS$_d$/CFT$_{d-1}$ correspondence (Witten, 1998), Savonije and Verlinde proved that the Cardy-Verlinde formula (15) can be derived using the thermodynamics of AdS-Schwarzschild black holes in arbitrary dimension (Savonije and Verlinde, 2001).

4. NON-COMMUTATIVITY AND ACHÚCARRO-ORTIZ BLACK HOLES

We consider a specific choice of spacetime non-commutativity as following

$$[x_i, x_j] = i\theta_{ij}, \qquad (i, j = 0, 1), \quad (x_0 = t, x_1 = x), \tag{20}$$

We have the following metric for Achúcarro-Ortiz black hole in non-commutative space

$$ds^2 = -g(\tilde{r})dt^2 + \frac{d\tilde{r}d\tilde{r}}{g(\tilde{r})} \tag{21}$$

with

$$g(\tilde{r}) = \left(-M + \frac{\tilde{r}\tilde{r}}{l^2} + \frac{J^2}{4\tilde{r}\tilde{r}}\right) \tag{22}$$

where $\tilde{r}$ satisfies following Poisson brackets (Li, 2005; Nasseri, 2005; Setare, 2006, accepted for publication)

$$\{\tilde{x}_i, \tilde{x}_j\} = \theta_{ij}, \quad \{\tilde{x}_i, \tilde{p}_j\} = \delta_{ij}, \qquad \{\tilde{p}_i, \tilde{p}_j\} = 0. \tag{23}$$

Since the non-commutativity parameter should very small compared to the length scales of the black hole, one can treat the noncommutative effects as some perturbations of the commutative counter-part, $g(\tilde{r})$ in terms of the noncommutative coordinates $\tilde{x}_i$ is as

$$g(\tilde{r}) = \left(-M + \frac{\tilde{x}_i\tilde{x}_i}{l^2} + \frac{J^2}{4\tilde{x}_i\tilde{x}_i}\right) \tag{24}$$

We note that there is a new coordinate system (Chaichian *et al.*, 2001)

$$x_i = \tilde{x}_i + \frac{1}{2}\theta_{ij}\tilde{p}_j, \qquad p_j = \tilde{p}_j, \tag{25}$$

where the new variables satisfy the usual Poisson brackets

$$\{x_i, x_j\} = 0, \quad \{x_i, p_j\} = \delta_{ij}, \quad \{p_i, p_j\} = 0. \tag{26}$$

Using the new coordinates, we have

$$f(r) = -M + \frac{\left(x_i - \frac{\theta_{ij}p_j}{2}\right)\left(x_i - \frac{\theta_{ik}p_k}{2}\right)}{l^2} + \frac{J^2}{4\left(x_i - \frac{\theta_{ij}p_j}{2}\right)\left(x_i - \frac{\theta_{ik}p_k}{2}\right)} \tag{27}$$

where $\theta_{ij} = 1/2\varepsilon_{ijk}\theta_k$. The equation

$$f(r_{h\pm}) = 0, \tag{28}$$

where $r_{h\pm}$ is the modified horizon, leads us to

$$f(r_{h\pm}) = -M + \frac{r_{h\pm}^2 - \frac{\vec{L}.\vec{\theta}}{4} + \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{16}}{l^2} + \frac{J^2}{4\left(r_{h\pm}^2 - \frac{\vec{L}.\vec{\theta}}{4} + \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{16}\right)} = 0 \tag{29}$$

where $L_k = \varepsilon_{ijk}x_i p_j$, $p^2 = \vec{p}.\vec{p}$ and $\theta^2 = \vec{\theta}.\vec{\theta}$. We can rewrite Eq. (29) as

$$r^4 - \left(Ml^2 - \frac{\vec{L}.\vec{\theta}}{2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8}\right) \times r^2 - \left(Ml^2\vec{L}.\vec{\theta} + Ml^2\frac{(P^2\theta^2 + (\vec{P}.\vec{\theta})^2)}{16} - \frac{(\vec{L}.\vec{\theta})^2}{16}\right) = 0 \tag{30}$$

therefore the horizons of Achúcarro-Ortiz black hole in non-commutative space are as following

$$r_{h\pm} = l\left[\frac{M}{2}\left(1 - \frac{\vec{L}.\vec{\theta}}{Ml^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8Ml^2} \pm \sqrt{1 - \left(\frac{J}{Ml}\right)^2}\right)\right]^{1/2} \tag{31}$$

For rotating Achúcarro-Ortiz black hole, in the limit $\theta \to 0$, the horizons are reduced to one in commutative space. For the non-rotating Achúcarro-Ortiz black hole, the horizons are as following

$$r_{h+} = l\left[M - \frac{p^2\theta^2 - (\vec{P}.\vec{\theta})^2}{16l^2}\right]^{1/2} \tag{32}$$

$$r_{h-} = \frac{[p^2\theta^2 - (\vec{P}.\vec{\theta})^2]^{1/2}}{4} \tag{33}$$

It is interesting that in non-commutative case, non-rotating Achúcarro-Ortiz black hole in contrast with commutative case has two horizons. As one can see in non-rotating case, to the first order of perturbative calculations, there is no any effect on horizon r_+ due to the non-commutativity of space, and $r_- = 0$.

The horizon area, Hawking temperature and entropy of Achúcarro-Ortiz black hole in non-commutative space are respectively as

$$A = 2\pi r_{h+} = 2\pi l \left[\frac{M}{2} \left(1 - \frac{\vec{L}.\vec{\theta}}{Ml^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8Ml^2} + \sqrt{1 - \left(\frac{J}{Ml}\right)^2} \right) \right]^{1/2} \tag{34}$$

$$T_H = \frac{1}{2\pi l^2} \left(\frac{r_{h+}^2 - r_{h-}^2}{r_{h+}} \right) = \frac{\sqrt{1 - \left(\frac{J}{Ml}\right)^2}}{\pi \left[\frac{M}{2} \left(1 - \frac{\vec{L}.\vec{\theta}}{Ml^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8Ml^2} + \sqrt{1 - \left(\frac{J}{Ml}\right)^2} \right) \right]^{1/2}} \tag{35}$$

$$S_{bh} = 4\pi l \left[M - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{16l^2} \right]^{1/2} \tag{36}$$

5. SPACE NON-COMMUTATIVITY CORRECTIONS TO THE CARDY-VERLINDE FORMULA

In this section we compute the effect of space non-commutativity to the entropy of a Achúcarro-Ortiz black hole described by the Cardy-Verlinde formula (15). It is easily seen from Eqs. (9) and (13) that

$$T_H S_{bh} = 2 \left(\frac{r_+^2}{l^2} - \frac{J^2}{4r_+^2} \right) \tag{37}$$

while from Eqs. (7) and (8) we have

$$\Omega_+ J = \frac{J^2}{2r_+^2} \tag{38}$$

Since the two-dimensional Achúcarro-Ortiz black hole is asymptotically anti-de-Sitter, the total energy is $E = M$ and thus the Casimir energy, substituting Eqs. (6), (37) and (38) in Eq. (16), is given as

$$E_C = \frac{J^2}{2r_+^2} \tag{39}$$

where in our analysis the charge Q is the angular momentum J of the two-dimensional Achúcarro-Ortiz black hole, the corresponding electric potential ϕ is the angular velocity Ω, and $n = 1$. The energy Eq. (6) and Casimir energy Eq. (39) now will be modified as

$$E = M = \frac{r_{h+}^2 + r_{h-}^2}{l^2} = M\left(1 - \frac{\vec{L}.\vec{\theta}}{Ml^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8Ml^2}\right)$$

$$= E_0\left(1 - \frac{\vec{L}.\vec{\theta}}{Ml^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8Ml^2}\right), \tag{40}$$

$$E_C = \frac{J^2}{2r_{h+}^2} = \frac{2r_{h-}^2}{l^2} = M\left(1 - \frac{\vec{L}.\vec{\theta}}{l^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8Ml^2} + \sqrt{1 - \left(\frac{J}{Ml}\right)^2}\right)$$

$$= E_{C0} - \left(\frac{\vec{L}.\vec{\theta}}{l^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8l^2}\right) \tag{41}$$

$$S_{CFT} = \frac{2\pi R}{\sqrt{ab}}\left[\left[E_{C0} - \left(\frac{\vec{L}.\vec{\theta}}{l^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8l^2}\right)\right]\right.$$
$$\times\left[2E_0\left(\frac{\vec{L}.\vec{\theta}}{Ml^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8Ml^2}\right) - E_{C0}\right.$$
$$\left.\left.+\frac{\vec{L}.\vec{\theta}}{l^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8l^2}\right)\right]\Bigg]^{1/2}$$

$$\simeq S_{0CFT}\left[1 - \frac{E_0(E_{C0}+1)}{(2E_0 - E_{C0})E_{C0}}\left(\frac{\vec{L}.\vec{\theta}}{l^2} - \frac{P^2\theta^2 - (\vec{P}.\vec{\theta})^2}{8l^2}\right) + \dots\right] \tag{42}$$

6. CONCLUSION

In this paper we have examined the effects of the space non-commutativity on the properties of rotating and non-rotating Achúcarro-Ortiz black holes, then we applied the results on the Cardy-Verlinde formula. The event horizon of the black hole undergoes corrections from the non-commutativity of space as Eq. (31) for rotating case, and Eqs. (32) and (33) for non-rotating case. It is interesting that in non-commutative case, non-rotating Achúcarro-Ortiz black hole in contrast with commutative case has two horizons. Since the non-commutativity parameter is so small in comparison with the length scales of the system, one can consider the noncommutative effect as perturbations of the commutative counterpart (Li, 2005; Nasseri, 2005; Setare, 2006, accepted for publication). Then we have derived the corresponding correction to the Cardy-Verlinde formula which relates the entropy of a certain CFT to its total energy and Casimir energy. The result of this paper is that the CFT entropy can be written in the form (42). It is necessary to mention that, our result in the present paper is valid for a specific choice of spacetime non-commutativity which is defined by Eq. (20). To see a more general kind of spacetime non-commutativity refer to (Mendes, 1994; Chryssomalakos and Okon, 2004; Ahluwalia, 2005a,b), in these papers, the principle of Lie algebra stability of the Poincare-Heisenberg algebra leads to a more general kind of spacetime non-commutativity. In those modifications, the commutators of spacetime coordinates is given by the generators of rotations and boosts.

Also one can consider the situation as the present paper with logarithmic corrections in $SAdS_5$ or in SdS_5 bulk backgrounds (Nojiri *et al.*, 2003), we hope to come back at future to this important problem.

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